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Probabilistic Uncertainty and Risk Management in Optimal Power Flow Distribution System

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Abstract: *This research focuses on optimal power flow based on probability-constrained, for limiting the constraint violation chances. The proposed method estimate to model policy-based mechanism and management actions in response to uncertainty realizations, that encourage to control and mitigate the possible adverse impacts. Power flow analysis and experiments are carried out for verification of electrical power transfer from generators to user through the grid system stability, reliability and economics. The simulation is presented on the IEEE 5-bus system, for the generator bids for power and reserves, the linear cost coefficients are applied. At probabilistic threshold, the results represent the optimal power flow at various level of risk and various level of incremental operational cost.*

Keywords: *Optimal Power Flow, Probabilistic Uncertainty, Risk Assessment, Risk Analysis, IEEE 5-bus system*

I Introduction

Optimal power flow (OPF) is one of the important static power system calculations [1, 2]. It has broad application in electrical engineering, including scheduling of generators, loss reduction, congestion management, and expansion planning. As distributed generation (DG) and controllable loads (e.g., electric vehicles) proliferate, active network management has been introduced in distribution systems [3]. The, optimal power flow (OPF) is no longer limited to the domain of high voltage transmission networks and has been gradually investigated for application to distribution networks [4, 5]. The fundamental model of power flow in distributed network is represented in Figure 1.

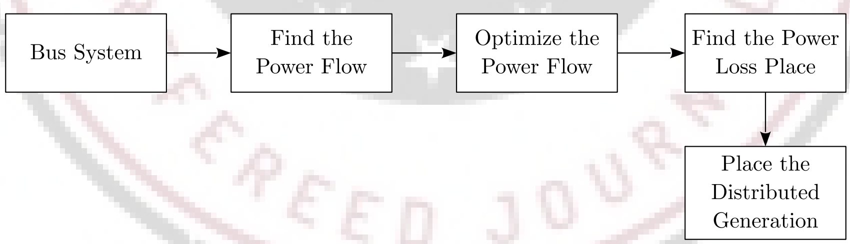

Figure 1: Basic Model of Power Flow in Distributed Network

In general, all the input data of optimal power flow (OPF) are deterministic. Governed by nonlinear Kirchhoff's laws, such deterministic optimization problems can be solved by many methods, such as successive linear/quadratic programming [6, 7], trust-region-based methods [8], the Lagrangian Newton method [9] and the interior-point method [10–12]. However, with increasing internal and external factors of uncertainty, such as the power demand affected by daily economic activities, power generated by renewable energy, and grid parameters obtained by approximate measurements, the input data have increasing uncertainty, which challenges conventional deterministic optimal power flow (OPF) models. The degree of uncertainty for some factors can be reduced, but for most uncontrollable factors, it is very difficult to decrease the impact of their uncertainty. Hence, optimal power flow (OPF) should be able to manage uncertainties in power flow performance [13–15].

Most conventional methodologies to address uncertainty are based on probabilistic methods that account for the variability and stochastic nature of the input data. Current Optimal Power Flow (OPF) research on this topic can be divided into two categories, probabilistic OPF (P-OPF) and stochastic OPF (S-OPF) [16, 17]. P-OPF is a well-respected approach for characterizing the output of an implicit function whose inputs are random variables, where the cumulant method [18] and the point estimate method [19] are examples of very efficient P-OPF computation. However, the solution of P-OPF is influenced indirectly by the randomness of input variables, and only the probability distributions of control variables can be determined. In S-OPF problems, the objective function and constraints are usually described by probability equations or inequalities, which means that the randomness of input variables can directly impact the solution [20, 21]. Thus, constraint satisfaction in an uncertain environment can be achieved.

II Related Work

II-A Risk Analysis for Shanghai's Electric Power System under Multiple Uncertainties

Piao *et al.* [22] developed a robust interval-fuzzy programming (RIFP) approach for planning electric power systems (EPS). RIFP can deal with multiple uncertainties expressed as fuzzy-boundary intervals and probability distributions, but also provide an effective linkage between the pre-regulated policies and the associated corrective actions against any in-feasibility arising from random outcomes. Then, a RIFP-based municipal-scale electric-power-systems planning (RIFP-MEP) model is formulated for Shanghai's EPS to demonstrate its applicability. With the aid of RIFP-MEP model, solutions under different feasibility degrees have been obtained for supporting the city's energy supply, electricity generation, conversion, transmission, utilization, facility-expansion schemes as well as air pollution control. The results can be used to make compromises among system cost, satisfaction degree, and constraint-violation risk.

The robust interval-fuzzy programming (RIFP) method shows more robust capacities in reflecting multiple uncertainties and examining system-failure risks through two recourse actions. One of the recourse actions is launched to capture the notion of risk in stochastic programming; another one is seized the risk of electricity shortage particularly when energy demand is high.

II-A.1 Robust Optimization

Robust optimization could not only penalize the costs that are above the expected values, but also capture the notion of risk under uncertainty [23]. In fact, the Robust Optimization (RO) method is a hybrid of stochastic and goal programs, to balance the tradeoff between the expected recourse costs and the variability of these random values [24]. A general Robust Optimization (RO) model [25] can be formulated as follows:

$$\min f = C_{T_1}X + \sum_{h=1}^{S} p_h D_{T_2}Y + \rho \sum_{h=1}^{S} p_h \left(D_{T_2}Y - p_h \sum_{h}^{S} D_{T_2}Y + 2\theta_h \right) \tag{1}$$

In the above modeling formulation, the random variables take discrete values with probability levels p_h, where $h = 1, 2, \ldots, s$ and $\sum p_h = 1$. The x_j and y_{jh} represent the first and second-stage decision variables, respectively; the term of $\left(D_{T_2}Y - p_h \sum_{h}^{S} D_{T_2}Y + 2\theta_h \right)$ is a variability measure on the second-stage penalty costs; the nonnegative factor ρ represents a weight coefficient; the θ_h is slack variable used for attaining looser constraints. Depending on the value of ρ, the optimization may favor solutions with a higher expected second-stage cost $\sum_{h=1}^{S} p_h D_{T_2}Y$ in exchanging for a lower variability in the second-stage penalty costs as measured by $\left(D_{T_2}Y - p_h \sum_{h}^{S} D_{T_2}Y + 2\theta_h \right)$ [26]. When $\rho = 0$, the Robust Optimization (RO) model becomes a conventional two-stage stochastic programming (TSP) one (i.e. the objective is only to minimize the first and second-stage costs); this also implies that the decision makers possess a risk neutral attitude and would not consider the variability of the uncertain recourse costs. However, when $\rho = 1$, the decision makers can consider the variability of the second-stage cost based on a risk-aversive attitude.

II-B Risk Assessment for Power System Operation Planning With High Wind Power Penetration

Negnevitsky *et al.* [27] presented a novel risk-assessment approach to quantitative evaluation of the security of a wind integrated power system for short-term operation planning. A risk index representing both the likelihood and consequences of contingencies as well as system uncertainties caused by wind power generation (WPG) and load forecasting errors was used to assess the system security. In the case study presented in the paper, the proposed approach was used to evaluate operational risks of the nine-bus power system with characteristics similar to the Tasmanian power system. The results showed that the integration of WPG significantly affected the system operational risk, especially risks associated with frequency response inadequacy.

Impacts of different factors including load and wind power generation (WPG) forecasting uncertainties, wind power penetration levels, and operating reserves on the system security were investigated. It also showed that the proposed approach could assist system operators in operation planning such as setting constraints for wind generation curtailments and determining operating reserves. One of the main contributions of this paper is the development of the analytical method for evaluating risks associated with frequency response inadequacy so that the frequency response adequacy assessment can be performed simultaneously with the steady-state voltage and overload evaluations in the system operational risk assessment.

II-B.1 Risk Assessment for Wind Integrated Power Systems

To perform power system risk assessments, a risk index needs to be defined. Since risk refers to "the effect of uncertainty on objective" and is often expressed in terms of a combination of the consequences of an event and the associated likelihood of occurrence [28], a risk index can be defined as the sum of products of probabilities and quantified consequences as

$$\text{Risk} = \sum_j \sum_i P(C_i) \times P(S_j) \times Q(C_i, S_j) \tag{2}$$

where,
$P(C_i)$ is the probability of the th contingency,
$C_i, P(S_j)$ is the probability of system operating condition,
S_j and $Q(C_i, S_j)$ is the quantified consequence of the contingency C_i in the operating condition S_j.

An operating condition includes different components, such as load and generation levels, network configuration, and

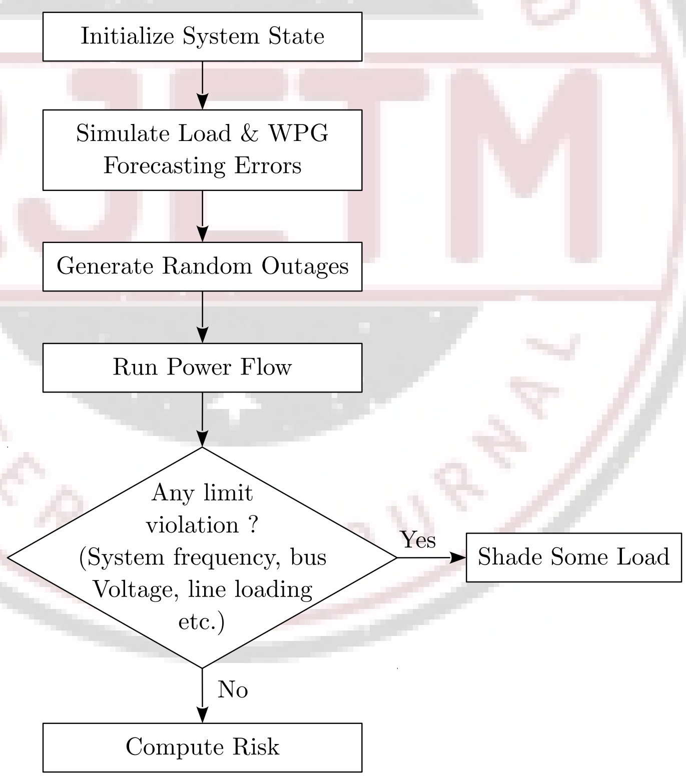

Figure 2: Simulation Procedure in Risk Assessment for Power System Operation Planning

possible operation measures. Some components are random with a specified probability distribution. The probability of the operating condition is based on the probability distribution of these components. The quantified consequence can be determined as the amounts of limit violation, load curtailments, or socio-economic losses due to contingencies, depending on the purpose of the risk assessment [29].

The simulations (steady-state power flow calculations) were conducted using DIgSILENT PowerFactory. The simulation procedure is presented in Figure 2. Generation-load imbalances are taken care by the generator inertial and governor responses.

If the system of interest is part of the large interconnected network, the lost generation will be picked up by a large number of generating units outside the system's immediate control area. In this case, the pickup in generation will appear as an increase in power flow over the tie-line. To represent this situation, we could build a model of our own network, add an equivalent model of the large neighboring system and place the swing bus in the equivalent system.

II-C Congestion Risk-Aware Unit Commitment with Significant Wind Power Generation

Abedi *et al.* [30] proposed a risk-aware Unit Commitment (UC) approach with the aim to reach a cost-effective real-time dispatch while improving the wind power utilization as well as alleviation of transmission congestion. Using the proposed Line Transfer Margins (LTM), the impact of multi-locational wind power uncertainty and correlation on the transmission capacity constraint in the UC problem is quantified. Thus, the proposed method can reduce wind integration and forecast error costs, as the day-ahead committed units and reserve resources are more diverse and power flow dispatch can fittingly follow the information regarding the uncertainty in wind power forecast by using Line Transfer Margins (LTM) signals. Demonstrated results and analysis confirm that using this approach outperforms the conventional Unit Commitment (UC) and Real-Time Dispatch (RTD) framework to enhance the reliability of power system operations with wind power integration, while leading to more cost-effective power system operation.

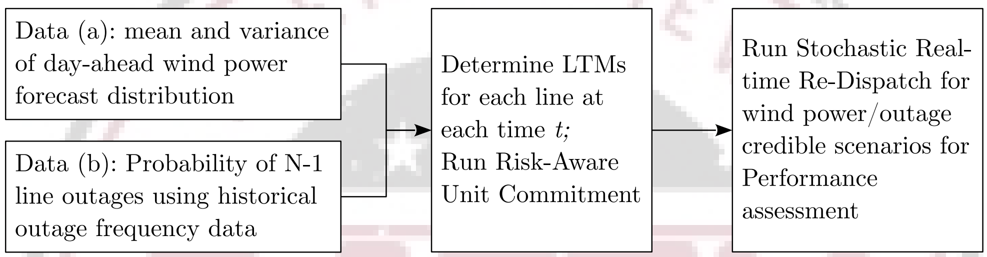

Figure 3: Flowchart of conducting Risk-Aware Unit Commitment (UC) with Line Transfer Margins (LTM)

The main purpose of conducting the real-time dispatch process in this work is to examine the efficiency of the proposed method in reducing the security violations under real-time operation of power system (as shown in Figure 3). Therefore, the real-time dispatch model herein should be inclusively capable of identification of infeasible cases incurred by the constraint violations, such as transmission congestion, in advance and provide corrective measures to treat them.

II-D Optimal Power Flow with Renewable Energy Resources including Storage

Reddy *et al.* [31] tackled the problem of optimal power flow with renewable energy resources and storage by taking into cognizance uncertainties in wind, solar PV power and load demands during real time. The anticipated real-time mean adjustment cost that accounts for the wind, solar PV power and load demand uncertainties, is introduced to accomplish this. Validation of results for a few cases has also been done using Monte Carlo simulation [32].

II-D.1 Real-Time Optimal Power Flow (RT-OPF) Model

Probabilistic real-time optimal power flow (RT-OPF) is used to calculate the mean adjustment cost (MAC), and the two-point estimate optimal power flow (OPF) is used to solve this RT-OPF problem. This two-point estimate method (2PEM) uses deterministic OPF. The deterministic and probabilistic RT-OPF models are formulated next.

In this model, the objective is to minimize the deterministic mean adjustment cost (MAC), and is formulated as,

$$\text{Minimize} \sum_{i=1}^{N_G} C_{\mathrm{RT}i}\left(P_{\mathrm{Dev}i}\right) = \sum_{i=1}^{N_G} C_{\mathrm{RT}i}\left(\left|P_{Gi}^{\mathrm{DA}} - P_{Gi}^{\mathrm{RT}}\right|\right) \tag{3}$$

II-E Transmission Line Overload Risk Assessment for Power Systems With Wind and Load-Power Generation Correlation

Li *et al.* [33] presented a method for assessing line overload risk for wind-integrated power systems with the consideration of wind and load-power generation correlations. Using point estimate method (PEM) for probabilistic load flow (PLF) calculations, the possibility of line overload can be computed. Combining the possibility with the severity of line overload, the quantitative risk indices can be obtained.

II-E.1 Probabilistic Model for Line Overload

The line overload possibility can be measured by the probability distribution of line flows. The probability distribution can be derived from the probabilistic load flow (PLF) calculation [27]. The output line-flow vectors Z can be expressed as a function of input random wind power, load, and power generation, as shown in Equation 4, where random vector y is composed of uncertain wind speed, loads, and power generations in a power system

$$Z = F(y) \tag{4}$$

The likelihood index in Equation 5 denotes the cumulative distribution function (CDF) of random variables whose samples do not satisfy the safety threshold with the confidence level t. With the assumption that line flows follow the normal distribution, t is the confidence level when random variable samples comply with the "3σ principle" as shown in Equation 6

$$\mathrm{Lik}(Z_l) = P_t\left(Z_l \notin [Z_{l\,\min}, Z_{l\,\max}]\right) \tag{5}$$

$$t = P\left(|Z_l - E(Z_l)| < 3\sqrt{D(Z_l)}\right) \tag{6}$$

The cumulative distribution function (CDF) of random variables can be written as Equation 7, where the probability distribution function (PDF) $f(Z_l)$ can be obtained from the results of the probabilistic load flow (PLF) calculation

$$P_t\left(Z_l \notin [Z_{l\,\min}, Z_{l\,\max}]\right) = \int_{Z_l \notin [Z_{l\,\min}, Z_{l\,\max}]} f(Z_l)dZ_l \tag{7}$$

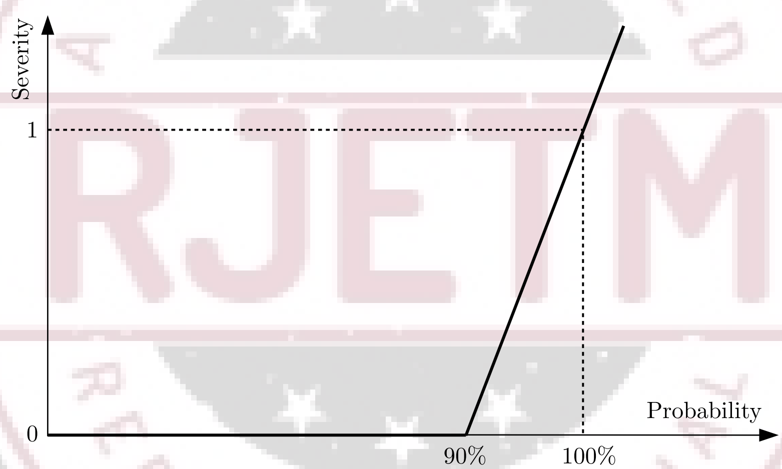

Figure 4: Severity Function of Line Overload

II-E.2 Severity Model of Line Overload

Severity function is used to uniformly quantify the severity of line overload. Severity function for line overload is related to the real power flow of a transmission line and is specified for each transmission line. Mean values of the real power flow as a percentage of the power rating would determine the overload severity of a line. The continuous severity function for line overload shown in Figure 4 is used in this research. Other severity functions, such as discrete severity functions, may also be used, and the proposed line overload risk assessment method is still valid [34].

II-E.3 Risk Index of Line Overload

The overload risk index $\mathrm{Risk}(Z_l)$ of line l Equation 8 is defined as the product of the probability $P_t(Z_l)$ and the severity $S_e(Z_l)$ of the transmission line overload. Therefore, the line overload risk index of the entire system can be calculated as Equation 9

$$\mathrm{Risk}(Z_l) = \int_{-\infty}^{+\infty} P_t(Z_l)S_e(Z_l)dZ_l \tag{8}$$

$$R_{\mathrm{all}} = \sum \mathrm{Risk}(Z_l) \tag{9}$$

III Proposed Approach

The purpose of an optimal power flow (OPF) is to schedule power system controls to optimize an objective function while satisfying a set of nonlinear equality/ and inequality constraints. Examples of these equality and inequality constraints include generation/load balance, bus voltage limits, power flow equations, branch flow limits (including both transmission line and transformer), active/reactive reserve limits, and limits on all control variables. The following is a simplified deterministic optimal power flow (OPF) problem with no discrete variables or controls. Risk modeling propose a risk measure for incorporation in an optimal power flow (OPF).

III-A Probabilistic Uncertainty and Risk Measures

A risk measure should reflect both the probability of an outage and the severity of the resulting operating condition. The risk related to a specific outage i and line k is expressed as

$$\mathcal{R}_{(i,k)}^{\text{spec}} := \mathcal{P}_{(i)} \cdot \mathcal{S}_{(k|i)} \tag{10}$$

where $\mathcal{P}_{(i)}$ is the probability of outage i and $\mathcal{S}_{(k|i)}$ is the severity of the operating condition on line k given outage i. This expression can be seen as the risk-based counterpart of the $N-1$ criterion, as it describes the risk for a specific line in one specific post-contingency state. Using $\mathcal{R}_{(i,k)}^{\text{spec}}$ as a basis, we define:

$$\mathcal{R}_{(i)}^{\text{out}} := \sum_{k=1}^{N_l} \mathcal{P}_{(i)} \cdot \mathcal{S}_{(k|i)} \tag{11}$$

$$\mathcal{R}_{(k)}^{\text{line}} := \sum_{i=1}^{N_{\text{out}}} \mathcal{P}_{(i)} \cdot \mathcal{S}_{(k|i)} \tag{12}$$

$$\mathcal{R}^{\text{tot}} := \sum_{i=1}^{N_{\text{out}}} \sum_{k=1}^{N_l} \mathcal{P}_{(i)} \cdot \mathcal{S}_{(k|i)} \tag{13}$$

$\mathcal{R}_{(i)}^{\text{out}}$ expresses the risk after an outage i, and is obtained by summing the risk of all lines k in this post-contingency state. $\mathcal{R}_{(k)}^{\text{line}}$ is the risk related to line k, summed over all outages i. $\mathcal{R}^{\text{tot}}$ is the total risk in the system, summed over all outages i and all lines k.

In order to evaluate Equation 10, the outage probabilities $\mathcal{P}_{(i)}$ must be estimated, and the severity $\mathcal{S}_{(k|i)}$ has to be defined. We assume that the outage probabilities are calculated a priori (e.g., based on historical data and current weather conditions [35]) and given as an input to the optimization.

III-B Severity Modeling

To capture different types of risk arising from different levels of post-contingency line loading, we define the severity $\mathcal{S}_{(k|i)}$ as a piece-wise linear function of the line flow. We define four different segments for the severity function. The four segments are, and correspond to

- Normal load,
- High load,
- Moderate overload which requires remedial actions,
- cascading overload which might lead to a cascading event

For the derivation of the severity function parameters, a system is considered the outage of any l line or m generator such that the set of contingencies $\mathcal{K} = \mathcal{L} \cup \mathcal{G}$. Although it is restricted itself to those $N-1$ outages, any other outage situations, e.g. $N-2$ or common mode outages, can be included without any change to the methodology. The post-contingency line flows on line ij after outage k is denoted by p_{ij}^k. Mathematically, it is defined as the piecewise linear severity function as the point-wise maximum over a set of affine functions of the line flow.

III-C Risk Constraints

Since the risk is modeled based on the post-contingency line flows, we can formulate risk-based constraints for these line flows. For other quantities, e.g. line flows in normal operation or generator outputs, no risk-based constraints can be formulated, since no risk level is defined for those quantities.

III-C.1 Formulation of Risk Constraints

Based on the risk measures defined by Equation 10–13, we can formulate constraints to limit the risk:

$$\mathcal{R}^{\text{spec}} := \mathcal{P}_{(i)} \cdot \mathcal{S}_{(k|i)} \left(P^i_{l(k)} \right) \leqslant \overline{\mathcal{R}}_{(i)} \tag{14}$$

$$\mathcal{R}^{\text{out}} := \sum_{i=1}^{N_{\text{out}}} \mathcal{P}_{(i)} \cdot \mathcal{S}_{(k|i)} \left(P^i_{l(k)} \right) \leqslant \overline{\mathcal{R}}^{\text{out}} \tag{15}$$

$$\mathcal{R}^{\text{line}} := \sum_{k=1}^{N_{N_l}} \mathcal{P}_{(i)} \cdot \mathcal{S}_{(k|i)} \left(P^i_{l(k)} \right) \leqslant \overline{\mathcal{R}}^{\text{line}} \tag{16}$$

$$\mathcal{R}^{\text{tot}} := \sum_{i=1}^{N_{\text{out}}} \sum_{k=1}^{N_l} \mathcal{P}_{(i)} \cdot \mathcal{S}_{(k|i)} \left(P^i_{l(k)} \right) \leqslant \overline{\mathcal{R}}^{\text{tot}} \tag{17}$$

Equation 14 constrains the risk for each line k to stay below a constant limit $\overline{\mathcal{R}}_i$ after the outage i. Equation 15 limits the risk of outage i, while Equation 16 limits the risk of line k and Equation 17 limits the total risk in the system.

IV Result Analysis

Power flow capabilities reduces the mean power output by a constant value and allows the power plants to provide reserves, and output control which enforces a hard threshold on the total power output. The corresponding control and reserve models were developed and incorporated into an optimal power flow formulation with probability constrains, which allows for controlling the risk of overloads due to wind fluctuations. Leveraging the convexity, the optimization problem probabilistic uncertainty is solved. Based on a simulation of the IEEE 5-bus system, power management and control with known risks can provide substantial cost benefits for power plants with high variability.

IV-A Optimal Power Flow Simulation Overview

For MATLAB simulation of power flow, the following assumption is considered.

- Power flow simulation analysis is very important in planning stages of new networks or addition to existing ones like adding new generator sites, meeting increase load demand and locating new transmission sites.
- The load flow solution gives the nodal voltages and phase angles and hence the power injection at all the buses and power flows through interconnecting power channels.
- It is helpful in determining the best location as well as optimal capacity of proposed generating station, substation and new lines.
- It determines the voltage of the buses. The voltage level at the certain buses must be kept within the closed tolerances.
- System transmission loss minimizes.
- Economic system operation with respect to fuel cost to generate all the power needed.
- The line flows can be known. The line should not be overloaded, it means, we should not operate the close to their stability or thermal limits.

Power flow analysis and studies are applied to ensure that electrical power transfer from generators to consumers through the grid system is economic, reliable and stable. There are many solution techniques for load flow analysis. The solution procedures and formulations can be precise or approximate, with values adjusted or unadjusted, intended for either on-line or off-line application, and designed for either single-case or multiple-case applications. Since an engineer is always concerned with the cost of products and services, the efficient optimum economic operation and planning of electric power generation system have always occupied an important position in the electric power industry.

IV-B Simulation Test on IEEE 5-Bus System

The MATLAB simulation is presented on the IEEE 5-bus system, with a few modifications as follows. For the generator bids for energy and reserves, the linear cost coefficients are used. For power plants, zero marginal cost is assumed. Although the formulation could be extended to include unit commitment, it is not considered here. Therefore, the minimum generation output of the conventional generators is set to zero. To obtain a more stressed system state, the load is increased by a factor of 1.25 and decrease the transmission limits by a factor of 0.75. Power plants are located at

Table 1: IEEE 5-Bus System Input Data

Bus	**Assumed**	**Generation**		**Load**	
Code	**Bus Voltage**	**MW**	**MVARs**	**MV**	**MVARs**
1	$1.06 + j0.0$	0	0	0	0
2	$1.0 + j0.0$	40	30	20	10
3	$1.0 + j0.0$	1	1	45	15
4	$1.0 + j0.0$	2	3	40	5
5	$1.0 + j0.0$	4	6	60	10

5 different buses throughout the system. The standard deviation of each wind power plant is set to 10% of the forecasted power output. When considering different levels of wind power penetration, both the forecasted power output and the standard deviations are scaled by a factor corresponding to the penetration of wind power relative to total system load. The risk limits are set to 0.1 MW. With the cutting-plane algorithm described above, a solution to the 5-bus system is obtained within seconds to control and manage power distribution. The Table 1 represents the input data assumed for IEEE 5-bus system to get generation and load output.

The normal procedure for a load flow study is to assume a balanced system and to use a single-phase representation equivalent to the positive sequence network. Since there is no mutual coupling, the bus admittance matrix can be formed by inspection and many of its elements will be zero. The experimental result from MATLAB simulation for magnitudes of bus voltages graph is represented in Figure 5, similarly the angle of bus voltages graph is represented in Figure 6.

Table 3 represents the output generated values in reference to the magnitude and angle of IEEE 5-bus voltages. The proposed allows not only to control the system risk level, but also to account for the effect of available remedial measures during the operational planning process. Through the use of probabilistic risk-based constraints, the post-contingency line flow limits are set based on which measures are available.

IV-C Probabilistic Risk Analysis

The contingency and line specific risk constraint $\mathcal{R}^{\text{spec}}_{(i,k)} \leq \overline{\mathcal{R}}$ is used for the risk-based formulations, with $\overline{\mathcal{R}} = \mathcal{R}^{\text{base}}$. For the probabilistic formulations, the maximum violation level was set to $\epsilon_J = 0.05$. Figure 7 represents the optimal power flow at different level of risk and different level of incremental operational cost and at probabilistic threshold. The resulting risk measure is used to formulate risk-based constraints for the post-contingency line flows. The optimal power flow formulation is applied to simulation analysis of the IEEE 5-bus system. Limit can be applied at different level of the risk even when the in-feeds deviate from the forecast. Further, the risk-based formulation allows to choose the desired risk level, for system security.

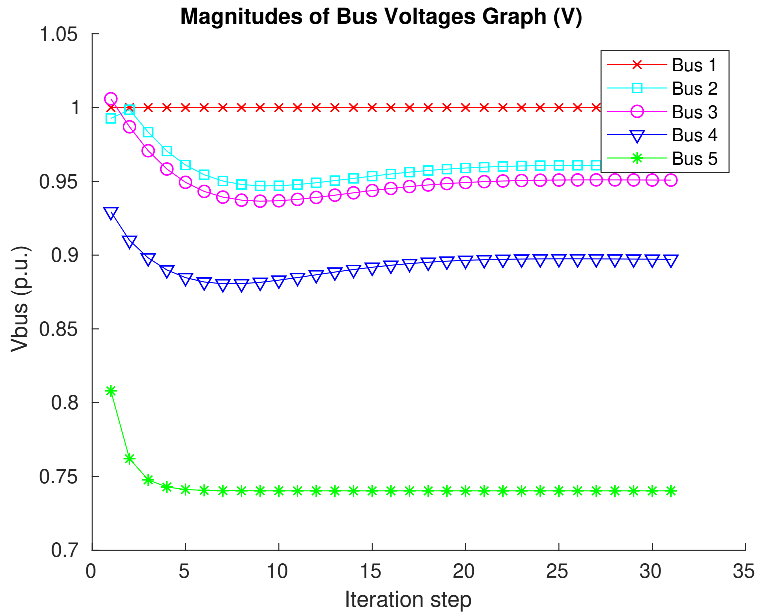

Figure 5: Magnitudes of Bus Voltages Graph

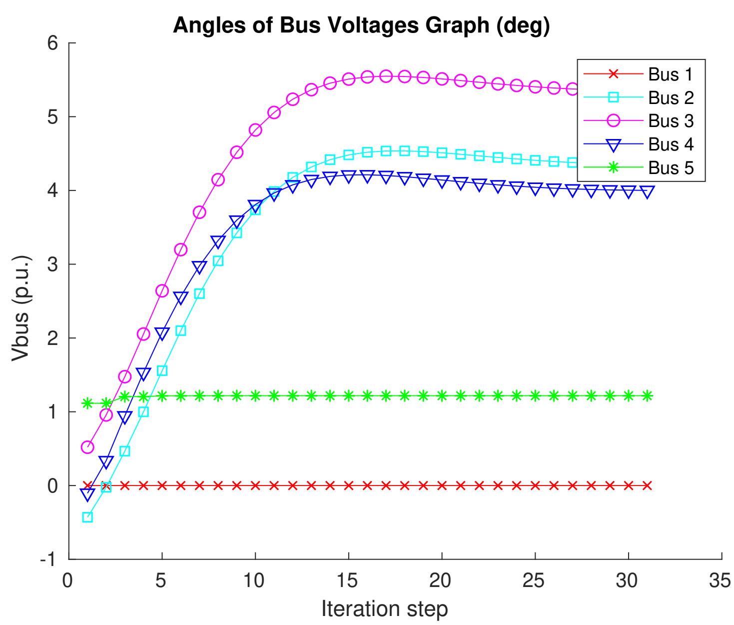

Figure 6: Angles of Bus Voltages Graph

V Conclusion and Future Work

In this paper, The IEEE 5-bus system is simulated and examined under assessment of probabilistic risk. There are various techniques to limit operational risk in optimal power flow problems where the constraints are affected by uncertainty. It is usually challenging to reformulate the stochastic optimization problem into a tractable, deterministic problem. The results represent the optimal power flow at various level of risk and various level of incremental operational cost at probabilistic threshold. Through the application of probabilistic risk-based constraints, the post-contingency line power flow limits can be set based on measures availability. The future scope for this work can be extended with Newton-Raphson method and Fast Decoupled methods.

Table 2: Output Power Generation

S.No.	Parameter	Output
1.	The active power generation of Slack bus	1.7639 MW
2.	The reactive power generation of Slack bus	1.7955 MVar
3.	The reactive generation $P-\|V\|$ bus	-2.5 MVar
4.	Number of iteration steps	31

Table 3: Magnitude and Angle Output of Bus Voltages

S.No.	Bus	Magnitude	Bus	Angle
1.	—V Bus 1—	57.3145 [V]	∠ Bus 1	0 deg
2.	—V Bus 2—	55.6488 [V]	∠ Bus 2	4.2922 deg
3.	—V Bus 3—	53.9986 [V]	∠ Bus 3	5.4232 deg
4.	—V Bus 4—	51.4502 [V]	∠ Bus 4	4.1675 deg
5.	—V Bus 5—	42.4267 [V]	∠ Bus 5	1.2213 deg

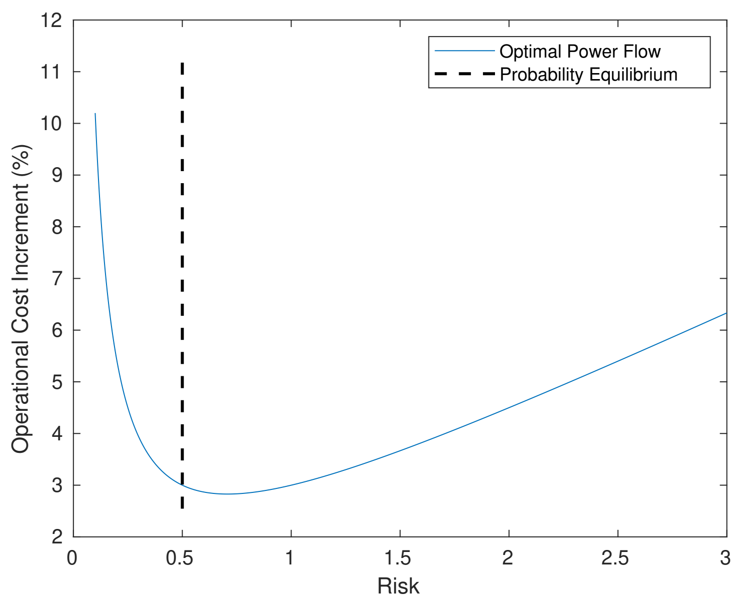

Figure 7: Optimal Power Flow at Different Level of Risks

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